Monitoring Vibration-based Structural Health Using Nonlinear Approach

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ABSTRACT

This paper presents a method for determining the degree of nonlinearity (DON) from the structural vibration data for monitoring the change in health of structures. Using Hilbert Transform of the frequency response function, the DON measures the nonlinearity present in the vibration response. It is shown that a plot of the DON against the magnitude of the motion represents the behavior of a structure. If the structure is damaged during a new striking motion, the DON parameter will deviate from its healthy signature. Data from numerical simulation and experimental measurement were used to evaluate the proposed method.

Keywords: Nonlinearity, Hilbert transform, Damage detection, Vibration-based health monitoring

1. INTRODUCTION

Until now, linear system identifications have been the backbone of the structural vibration-based health monitoring. A change in system parameters estimated by the linear model governing the vibration-based response indicates a change in the health of a structure [1]. For many civil engineering structures, this approach is questionable because the actual complicated vibration behavior may not be fully described by a simplified linear model. Also, the purpose of monitoring the health of a structure is to know its behavior. There is no reason to assume that the structure behaves linearly all the time for the sake of simplicity. Moreover, it is increasingly recognized that the damage of structures relates to multiple nonlinear behaviors such as the changes in material and/or geometric properties of the structural system, boundary conditions and system connectivity [2-5]. Therefore, the nonlinear parameter identification from the vibration response of a structure should be advantageous to the vibration-based health monitoring. This paper presents a method for measuring the degree of nonlinearity (DON) from the wireless acceleration response data and exploiting DON for early detection of damage in a two-story model test structure built on a shaking table. Theoretical computation and experimental validation of the DON parameter are presented in subsequent sections.

1.1 Theoretical Computation of Degree of Nonlinearity (DON)

When a nonlinear system is described by a linear system, it is to be sure that the best linear approximation is made. This follows that frequency response functions (FRFs), denoted by \( H(\omega) \), can be estimated as \( H(\omega) = S_{YU}(j\omega) / S_{UU}(j\omega) \) with \( S_{YU} \) the output-input cross spectrum and \( S_{UU} \) the input autospectrum. The Hilbert transform \( H^H(\omega) \) can also be carried out which allows one to calculate the imaginary part of the FRF from its real part and vice versa. In references [6-7], the inequality of the two quantities, \( H^H(\omega) \neq H(\omega) \), was employed as a criteria for detecting unmeasurable nonlinearity (type and severity) present in the response of the system. Assuming that the change in the vibration behavior from linear to nonlinear indicates damages in the system, the difference or the nonlinearity can be mathematically described in an approximate form of an unknown frequency dependent complex function \( \delta(\omega, u_{rms}) \) multiplied by the known function \( H(\omega) \) as

\[
H^H(\omega) - H(\omega) = \delta(\omega, u_{rms}) H(\omega)
\]  

(1)

where \( \omega \) is the frequency; \( u_{rms} \) is the root-mean-square magnitude of the input excitation. Equation (1) is expected to represent all frequency dependent characteristics of the type of the nonlinearity and input excitation.

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Integrating Eq.(1) over the frequency range of interest $[\omega_1, \omega_2]$ leads to

$$\int_{\omega_1}^{\omega_2} [H^{\alpha}(\omega) - H(\omega)] d\omega = \int_{\omega_1}^{\omega_2} |\delta(\omega, u_{rms})| H(\omega) d\omega$$

(2)

According to the mean value theorem, there exists a mean value $\bar{\delta}(u_{rms})$ of the unknown function $|\delta(\omega, u_{rms})|$ such that

$$\bar{\delta}(u_{rms}) = \frac{\int_{\omega_1}^{\omega_2} [H^{\alpha}(\omega) - H(\omega)] d\omega}{\int_{\omega_1}^{\omega_2} |H(\omega)| d\omega}$$

(3)

The mean $\bar{\delta}(u_{rms})$ in Eq.(3) is defined here as the degree of nonlinearity (DON) which simplifies and measures the unmeasurable nonlinearity in an average sense.

1.2 DON Characteristics and Its Use for Structural Health Monitoring

Using Eq. (3), numerical calculations are carried out in this section to show that the DON can detect cubic and saturation nonlinearities for a given nonlinear system. Let the nonlinear system be described by

$$m\ddot{y} + c\dot{y} + f_s(y) = u(t)$$

(4)

where $y$ is the response; $u$ is the uniform input random noise excitation with the root-mean-square average $u_{rms}$; the system parameters are $m = 1$, $c = 0.1$, and $k = 25.27$. From the nonlinear term $f_s(y)$, two types of nonlinearities are simulated. The cubic nonlinearity is governed by the condition $f_s(y) = ky + \alpha y^3$ where the nonlinear parameter $\alpha = 2.527$ (for low nonlinearity) and $\alpha = 12.64$ (for high nonlinearity). On the other hand, the saturation nonlinearity is governed by the condition $f_s(y) = -f_{max} < ky < f_{max}$ where the nonlinear parameter $f_{max} = 6.208$ (for low nonlinearity) and $f_{max} = 4.319$ (for high nonlinearity).

The simulation of both nonlinearities has been performed by varying the magnitude of the input excitation $u_{rms}$ from a small to large value. The simulated time histories of the excitation and response are used to compute DON. The plots of DON against $u_{rms}$ for the two cases are shown in Figure 1.

![Signature plot of DON vs. $u_{rms}$ for different types and severity levels of nonlinearity present in the system.](image)

Figure 1 shows that the DON computed by Eq.(3) depends only on the magnitude of the excitation. In case of a healthy system behaving linearly, the nearly constant or zero baseline plot of DON against $u_{rms}$ may be generated by collecting several sets of vibration response data. However, when the magnitude of the excitation exceeds a certain level at which the system becomes nonlinear, the significant deviation from the baseline values could be observed. It is also obvious from Figure 1 that plotting DON against $u_{rms}$ for a specific type of nonlinearity will produce a unique curve. Therefore, for a structure possessing certain characteristics of the nonlinearity, the plot of DON against $u_{rms}$ is considered as the “signature plot” of the structure. Basic properties of the plot including curvature and size make it an ideal means for early stage damage detection of structures under random input excitation. Any significant deviation from the baseline values indicates that the structure may become nonlinear, thereby indicating damages present in the system.
In conclusion, these results provide the evidence that the DON can measure nonlinearity based on the arbitrary selection of a threshold. The signature plot can then be used to detect the change in health or damage quickly without trying to first understand the underlying cause.

2. EXPERIMENTAL VALIDATION

The discussion in the previous section was carried out based on the simulated data. To confirm the validity and the applicability of DON, the output response of wireless sensors attached to a two-story model test structure was recorded in a series of shaking table tests for further analysis.

2.1 Test Setup

Figure 2 shows the two-story test structure built on a shaking table that produced random excitation at frequencies of 0 to 10 Hz. The model structure was constructed with aluminum for columns and steel for beams. Xbee-ADXL330 wireless accelerometer sensors were attached to each floor of the test structure, spaced at 30cm apart, to measure acceleration in three main directions: X-longitudinal, Y-transversal, and Z-vertical direction. The acceleration signals from all sensor units were transmitted to the base station connected to the PC at 200 Hz for 50 seconds.

![Figure 2. Experiment setup: test structure, shaking table, Xbee wireless acceleration sensor, PC with base station.](image)

A total of ten runs of shaking table tests were conducted on the healthy structure. Then another ten runs were performed when damage was introduced by loosening two of the three bolts constituting the connection between the beam and column members. The damage was identified and confirmed by visual inspection. The major shake table excitation was in X- direction. For each test run, the amplitude of the excitation was increased. Since the acceleration response data could be collected from the wireless monitoring module, the Fourier amplitude spectrum for each level was constructed thereafter immediately. Figure 3 shows plots of base and 2nd floor acceleration time history data measured at different level of structural damage during the first shake-table run. Figure 4 shows the Fourier amplitude spectrum computed from the responses shown in Figure 3.

![Figure 3. Plots of base and 2nd floor acceleration time history data measured from (a) healthy; (b) damaged structure.](image)
2.2 DON results and discussions

The average values of DON from all the sensors placed on the floors were computed and plotted against the magnitude of all the shake-table runs, as shown in Figure 5. It is apparent from Figure 5 that DONs were almost constant over the range of the magnitude of the measured accelerations on the shake-table for the healthy model test structure. As the structure was damaged - the reduction in stiffness resulted in change or decrease in the natural frequencies (Figure 4), DON increased in size. This was more pronounced when the magnitude of the excitation increased. This tendency was similar to the signature plot which appeared earlier in Figure 1. The severity of the nonlinearity had an obvious effect on the rate of increase in the DON at which the system became more nonlinear. These original results confirmed the validity of the DON.

2.3 DON for vibration based health monitoring

These results demonstrated and confirmed that DON could detect the change in the health of structure. The DON measures whatever nonlinearities into a single number thus overcame the limitations of linear and nonlinear system identification. The computation of the DON requires only the frequency response function (FRF) which could be obtained easily when the monitored ground motion and vibration response data are available. The signature plot generated for the healthy structure can serve as the baseline. The new DON that was statistically higher than the baseline values indicated that the structure behaved more nonlinearly or the characteristics of the nonlinearity had changed. Nonlinearity implies the change in the health of the structure. When this method is applied to practical situations, judgment criteria of DON should be investigated with wide range and long time data. The comparison of DON among different sensor locations will also be useful for that purpose.
3. SUMMARY AND CONCLUSIONS

The objective of this paper was to investigate the use of the degree of nonlinearity (DON) for vibration based health monitoring. The procedure consisted of extracting the feature from the experimentally monitored vibration data during shake table tests, and exploiting the feature to detect damage. The validity of DON was verified by two sets of data: 1) the simulated data from nonlinear systems and 2) the monitored data from the two-story test structure built on a shaking table. The simulated data was used to verify that the DON could represent the nonlinear behavior while the experimentally monitored data was used to verify that DON could detect damage. The scope of this paper is, however, limited to the early stage damage detection of structures. The information on aspects such as the location, types and severity of damage could be studied further in future depending upon the instrumentation for monitoring the experimental data.

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